Solution by the PROPOSER.

Let the equation of the parabola be $y^2=4px$, and that of the ellipse

$$\frac{(y-\beta)^2}{b^2}+\frac{(x-\alpha)^2}{a^2}=1.$$

A normal to the ellipse expressed by the tangent of the angle it makes with the axis of x has the equation

$$y-\beta=m(x-\alpha)-\frac{(a^2-b^2)m}{\sqrt{(b^2m^2+a^2)}},$$

and in the case of the parabola $y=mx-2pm-pm^3$. Since the shortest distance is measured off on a common normal, the two equations should be identical, and therefore

$$\beta-m\alpha-\frac{(a^2-b^2)m}{\sqrt{(b^2m^2+a^2)}}=-2pm-pm^3.$$

From this equation m is to be found. It leads to an equation of the 8th degree, which for numerical values presents no difficulty.

Substituting now the equation of the normal in both the equations of the parabola and ellipse, we find the co-ordinates of the points of intersection. Denoting these by x', y', and x'', y'', we find for the shortest distance the expression $\sqrt{[(x'-x'')^2+(y'-y'')]^2}$.

The numerical equations given having a common value, x lying between 6 and 7, and intersecting, therefore present no suitable example.

Also solved by *G. B. M. ZERR.*

CALCULUS.

129. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Among all quadrilaterals inscribed in an ellipse, to determine that which contains the greatest area.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Walton furnishes a solution of this interesting problem, which for its elegance and simplicity I reproduce here.

Let the equation of the ellipse be $x^2/a^2+y^2/b^2=1$, and let the angular points of the quadrilateral be (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4). Then, u denoting the area of the quadrilateral,

$$2u=x_2y_1-x_1y_2+x_3y_2-x_2y_3+x_4y_3-x_3y_4+x_1y_4-x_4y_1.$$

$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1,\quad \frac{x_2^2}{a^2}+\frac{y_2^2}{b^2}=1,\quad \frac{x_3^2}{a^2}+\frac{y_3^2}{b^2}=1,\quad \frac{x_4^2}{a^2}+\frac{y_4^2}{b^2}=1.$$

In order that u may be a maximum, we have, differentiating the last four equations, putting $du=0$, and using the method of indeterminate multipliers,

$$\frac{\lambda_1 x_1}{a^2}+y_4-y_2=0,\quad \frac{\lambda_1 y_1}{b^2}-x_4+x_2=0.$$

. . . . . .

Hence we have $\frac{x_1}{a^2}(x_4-x_2)=-\frac{y_1}{b^2}(y_4-y_2)....(1)$,

$$\frac{x_2}{a^2}(x_1-x_3)=-\frac{y_2}{b^2}(y_1-y_3)....(2),\quad \frac{x_3}{a^2}(x_2-x_4)=-\frac{y_3}{b^2}(y_1-y_4)....(3),$$

$$\frac{x_4}{a^2}(x_3-x_1)=-\frac{y_4}{b^2}(y_3-y_1)....(4).$$

From (1) and (3) we have $\frac{y_3}{x_3}=\frac{y_1}{x_1}....(5)$, and from (2) and (4), $\frac{y_4}{x_4}=\frac{y_2}{x_2}....(6)$. Also from (1) and (2), $\frac{1}{a^2}(x_1x_4-x_2x_3)=-\frac{1}{b^2}(y_1y_4-y_2y_3)$.

From the last three equations we see that

$$\frac{1}{a^2}(x_1x_4-x_2x_3)=-\frac{1}{b^2}y_1y_2\left[\frac{x_4}{x_2}-\frac{x_3}{x_1}\right], \text{ and therefore } \frac{y_1y_2}{x_1x_2}=-\frac{b^2}{a^2}..(7).$$

The equations (5) and (6) show that the diagonals of the quadrilaterals are diameters of the ellipse, and (7) shows that they are conjugate diameters.

Also solved by *G. B. M. ZERR*, *A. H. HOLMES*, and *L. C. WALKER*.

130. Proposed by J. SCHEFFER. A. M., Hagerstown, Md.

Solve the differential equation $x^x(\frac{dy}{dx}+y\log x)-a=0$.

I. Solution by W. E. HEAL, Marion, Ind.

Writing the equation $\frac{dy}{dx}+y\log x=a/x^x$, we have a linear equation of the standard form, and by the well known formula, $y=-Ca/x^x$.

II. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Dividing by x^x and transposing we get $dy/dx+\log x.y=ax^{-x}$, the regular form of the *linear* equation.

Using Bernoulli's method, we proceed as follows:

Let $y=uz$; then $dy/dx=u\frac{dz}{dx}+z\frac{du}{dx}$, and the given equation becomes

$$u\frac{dz}{dx}+z(\frac{du}{dx}+u\log x)=ax^{-x}.$$